EXAM 2 IS TUESDAY IN QUIZ SECTION

Allowed:

1. A **Ti-30x IIS Calculator**
2. An 8.5 by 11 inch sheet of handwritten notes (front/back)
3. A pencil or black/blue pen

Covers: 3.1-3.6, 10.2, 3.9, 3.10, 4.1

Quick Review

This test is about two things:

A) **All** Derivative Methods
B) Immediate concepts and applications of derivatives

See my newsletters (and homework) for targeted practice problems on any of topic. I will take some problems directly from homework.

***Applications and Concepts*:**

1. Geometric Tangent Slope/Line Questions
2. Linear approximation
3. Related Rates!!
4. Finding Critical Numbers
5. Finding Absolute Max/Min

***Derivative Methods*:**

6. Power Rule: $\frac{d}{dx}(x^n) = nx^{n-1}$

7. Exponential Rules: $\frac{d}{dx}(e^x) = e^x$, $\frac{d}{dx}(a^x) = a^x \ln(a)$

8. Product, Quotient, Chain Rules!!

9. Trig derivatives:

$\frac{d}{dx}(\sin(x)) = \cos(x)$	$\frac{d}{dx}(\cos(x)) = -\sin(x)$
$\frac{d}{dx}(\tan(x)) = \sec^2(x)$	$\frac{d}{dx}(\cot(x)) = -\csc^2(x)$
$\frac{d}{dx}(\sec(x)) = \sec(x)\tan(x)$	$\frac{d}{dx}(\csc(x)) = -\csc(x)\cot(x)$

10. Parametric Equation Derivatives

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \text{slope of tangent}$$

$$\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} = \text{speed}$$

11. Implicit Differentiation!!

12. Inverse Trig derivatives

$\frac{d}{dx}(\sin^{-1}(x)) = \frac{1}{\sqrt{1-x^2}}$	$\frac{d}{dx}(\cos^{-1}(x)) = -\frac{1}{\sqrt{1-x^2}}$
$\frac{d}{dx}(\tan^{-1}(x)) = \frac{1}{1+x^2}$	$\frac{d}{dx}(\cot^{-1}(x)) = -\frac{1}{1+x^2}$
$\frac{d}{dx}(\sec^{-1}(x)) = \frac{1}{x\sqrt{x^2-1}}$	$\frac{d}{dx}(\csc^{-1}(x)) = -\frac{1}{x\sqrt{x^2-1}}$

13. Deriv. of Logarithms: $\frac{d}{dx}(\ln(x)) = \frac{1}{x}$, $\frac{d}{dx}(\log_a(x)) = \frac{1}{x\ln(a)}$

14. Logarithmic Differentiation

Math 124 - Winter 2017
Exam 2
February 21, 2017

Name: _____

Section: _____

Student ID Number: _____

PAGE 1	14	
PAGE 2	12	
PAGE 3	14	
PAGE 4	8	
PAGE 5	12	
Total	60	

- There are 5 pages of questions. Make sure your exam contains all these questions.
- You are allowed to use a Ti-30x IIS Calculator model ONLY (**no other calculators allowed**). And you are allowed one **hand-written** 8.5 by 11 inch page of notes (front and back).
- Leave your answer in exact form. Simplify standard trig, inverse trig, natural logarithm, and root values. Here are several examples: you should write $\sqrt{4} = 2$ and $\cos\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$ and $\frac{7}{2} - \frac{3}{5} = \frac{29}{10}$ and $\ln(1) = 0$ and $\tan^{-1}(1) = \arctan(1) = \frac{\pi}{4}$.
- Show your work on all problems. The correct answer with no supporting work may result in no credit. **Put a box around your FINAL ANSWER for each problem and cross out any work that you don't want to be graded.**
- If you need more room, use backs of the pages and indicate to the grader that you have done so.
- Raise your hand if you have a question.
- There may be multiple versions of the exam so if you copy off a neighbor and put down the answers from another version we will know you cheated. Any student found engaging in academic misconduct will receive a score of 0 on this exam. All suspicious behavior will be reported to the student misconduct board.
  DO NOT CHEAT OR DO ANYTHING THAT LOOKS SUSPICIOUS!
  WE WILL REPORT YOU AND YOU MAY BE EXPELLED!
  Keep your eyes down and on your paper. If your TA sees your eyes wandering they will warn you only once before taking your exam from you.
- You have 80 minutes to complete the exam. Budget your time wisely.
  SPEND NO MORE THAN 10 MINUTES PER PAGE!

GOOD LUCK!

1. (14 pts) (You don't need to simplify your derivatives)

   (a) Let $y = \tan^5(e^{3x})$. Find $\frac{dy}{dx}$.

   (b) Let $g(x) = \dfrac{x}{2} + \arctan(2x)$.
   Find the value(s) of x at which the slope of the tangent line to $g(x)$ is 1.

   (c) Find the value(s) of x at which $f(x) = \sqrt{4x - x^{4/3}}$ has a horizontal tangent line.

2. (12 pts) For all parts below, consider a particle moving in the xy-plane such that its location at time t seconds is given by:

$$x(t) = \frac{1}{3}t^3 - \frac{7}{2}t^2 + 10t + 2 \quad , \quad y(t) = \ln(3t+1) + 4^t - \ln(4)t + 5,$$

where x and y are in feet.

(a) Find the following:

i. The formula for the vertical velocity in terms of time t.

ii. The speed of the particle at time $t = 0$. (include units)

iii. The equation for the tangent line to the curve at time $t = 0$ (in the form $y = mx + b$).

(b) Find all times, t, then the curve has a **vertical** tangent line.

3. (14 pts)

(a) Find the equation of the tangent line to $y = (2x+1)^{\cos(\pi x)}$ at $x = 1$.

(b) The implicit defined curve $(x^2+y)^2+xy^5 = 4$ has only one point where it crosses the positive y-axis. Find the equation of the tangent line at this positive y-intercept.

4. (8 pts) A spherical snowball is melting. At the moment when the radius is 5 cm, its surface area is decreasing at a rate of 3 cm^2/min. Find the rate at which the **volume** is changing at this same moment. (Include units in your final answer and indicate if your answer is positive or negative).

   Recall: Volume of a sphere is $V = \frac{4}{3}\pi r^3$ and the surface area of a sphere is $S = 4\pi r^2$.

5. (12 pts) A kite is in the air at an altitude of 400 feet and is being blown *horizontally* at the constant rate of 10 feet per second away from the person holding the kite string at ground level. (Thus, the kite is remaining at a constant altitude of 400 feet).
For both parts: Include units for your final answers and indicate if your answers are positive or negative. Your final answers should be simplified numbers/fractions.

   (a) At what rate is the string being let out when 500 feet of string is already out?

   (b) Let θ be the angle the string makes with the ground at a given time. At what rate is θ changing at the instant when 500 feet of string is already out?